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Strict and Defeasible Transitivities in Nonmonotonic Inheritances

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Strict and Defeasible Transitivities in Nonmonotonic Inheritances

(Extended Abstract)

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Abstract

The principle of inheritance is transitivity in hierarchical structuring knowledge. This paper presents a theory on nonmonotonic inheritance reasoning based on this principle. It is introduced two types of transitivities in inheritance hierarchy, strict and defeasible transitivities, which give simple and clear semantics for multiple inheritance with exceptions. Application to the classical problems is also discussed by comparison with other approaches.

Topic Area: Commonsense Reasoning

Keyword: Nonmonotonic Inheritance

1 Introduction

One of the most important areas in commonsense reasoning is inheritance hierarchies which enable us to represent information implicitly in hierarchical structuring knowledge. Inheritance is transitive in exception-free hierarchies but this principle does not hold in the presence of exceptions. The lack of a theory which gives sound and intuitive semantics is the reason for the recent competitive studies on nonmonotonic inheritances.

Early work by [Etherington83] has shown that such nonmonotonic hierarchies can be expressed using seminormal defaults in Reiter's default logic [Reiter80]. [Touretzky84] improved this formalization using normal default theory with ordering, and introduced a new rule called *inferential distance ordering* which reflects the intuition that subclasses should override superclasses in multiple inheritance with exceptions. [Sandewall86] has also given an alternative inheritance system which improved Touretzky's system, although some problem is still remained[Touretzky87]. Many studies have been proposed to improve the drawback of these theories; however, they tend to make the problem more complicated apart from the nature of inheritance, that is, transitive inference in structured knowledge.

Our approach in this paper is motivated to use such transitive inference in nonmonotonic inheritance hierarchies. We do so by introducing two types of transitivities in inheritance hierarchy, strict and defeasible transitivity. Using these transitivities, we give simple and clear semantics for nonmonotonic inheritance reasoning. We also apply our theory to the classical problems and compare it with other approaches.

2 Axioms

Let x be an individual (or a class) and y be a class. Then, $isa(x,y)$ (resp. $nisa(x,y)$) is read as "x is always a y (resp. always not a y)" and is called *strict isa*. While $isa^*(x,y)$ (resp. $nisa^*(x,y)$) is read as "x is normally a y (resp. normally not a y)" and is called *default isa*.[1] Now we give the axioms for inheritance in the following.

A1. $\forall\ x,y,z\ isa(x,y) \wedge isa(y,z) \supset isa(x,z)$

$\forall\ x,y,z\ isa(x,y) \wedge nisa(y,z) \supset nisa(x,z)$

A2. $\forall\ x,y,z\ isa(x,y) \wedge isa^*(y,z) \wedge M\ isa^*(x,z) \supset isa^*(x,z)$

[1] These notations are the same as [Etherington83], except for his exception link, while [Sandewall86] does not distinguish between these links.

$\forall$ x,y,z $isa^*(x,y) \wedge isa(y,z) \wedge M\ isa^*(x,z) \supset isa^*(x,z)$

A3. $\forall$ x,y,z $isa(x,y) \wedge nisa^*(y,z) \wedge M\ nisa^*(x,z) \supset nisa^*(x,z)$

$\forall$ x,y,z $isa^*(x,y) \wedge nisa(y,z) \wedge M\ nisa^*(x,z) \supset nisa^*(x,z)$

A4. $\forall$ x,y $isa(x,y) \wedge nisa(x,y) \equiv \perp$ (contradiction)

$\forall$ x,y $isa^*(x,y) \wedge nisa(x,y) \equiv \perp$

$\forall$ x,y $isa(x,y) \wedge nisa^*(x,y) \equiv \perp$

$\forall$ x,y $isa^*(x,y) \wedge nisa^*(x,y) \equiv \perp$

A1 defines transitive relations of strict *isa*, called *strict transitivities*. While A2 and A3, defining relations between strict and default *isa* using nonmonotonic modal operator M [McDermott80], are called *defeasible transitivities*.[2] A4 gives the conditions for contradiction.

Remarks

1. From modal aspect, $nisa(x,y)$ is not equal to $\neg isa(x,y)$, which is read as *x is not always a y*. This is also the case with $nisa^*(x,y)$ and $\neg isa^*(x,y)$.

2. We do not have the axioms:

   $\forall$ x,y,z $nisa(x,y) \wedge isa(y,z) \supset nisa(x,z)$,

   $\forall$ x,y,z $nisa(x,y) \wedge isa^*(y,z) \wedge M\ nisa^*(x,z) \supset nisa^*(x,z)$ and

   $\forall$ x,y,z $nisa^*(x,y) \wedge isa(y,z) \wedge M\ nisa^*(x,z) \supset nisa^*(x,z)$.

   This is because the transitivity does not hold through *nisa* to *isa*, in general. For example, *nisa(Mary,man)* and *isa(man,human)* should not yield *nisa(Mary,human)*. This is the case through *nisa* to *isa** and *nisa** to *isa*, too.[3]

3. The axoms do not permit transitivity for the default *isa* relation, that is, a derived fact containing *at most one* default *isa* in its derivation history. This is because the transitive chain for default *isa* reduces its correctness.[4]

[2] Strong and weak transitivities, different from ours, are discussed in the context of inheritable relations in [Touretzky86].

[3] [Touretzky86] also prohibits such kind of inheritances.

[4] Suppose $isa^*(x,y)$ ensures 80% correctness, then the transitivity $isa^*(x,y), isa^*(y,z) \rightarrow isa^*(x,z)$ reduces it to 64%, and further concatenation ensures less correctness [Sandewall86].

For example, consider the *penguin triangle.* In this case, a penguin is always a bird and not a flier, while a bird is normally a flier. We represent these facts by the following axioms:

isa(penguin,bird),
nisa(penguin,flier),
isa(bird,flier).*

In this case, *isa(penguin,bird)* and *isa*(bird,flier)* do not yield *isa*(penguin,flier)*, since *isa*(penguin,flier)* contradicts *nisa(penguin,flier)*, and axiom A2 blocks this implication.

Our theory is a natural extension of transitive inheritance in hierarchical structuring knowledge and gives simple and intuitive semantics for nonmonotonic inheritance hierarchies. Next we apply our theory to the classical problems.

3 Examples

(1) *Clyde's Color*

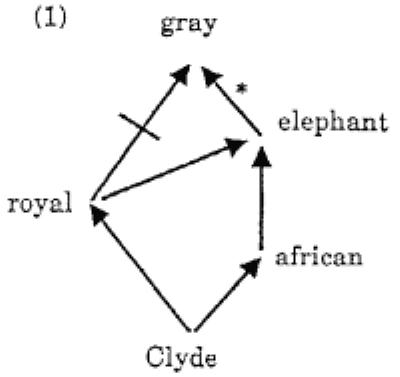
This is a typical example of multiple inheritance with exception. The following axioms give this hierarchy:

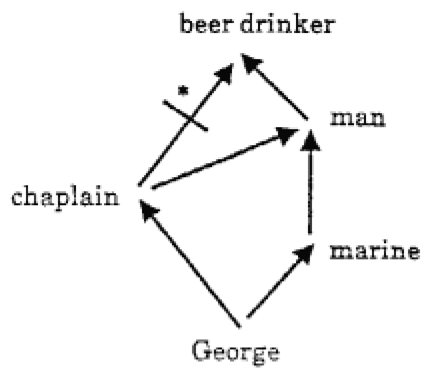
isa(Clyde,royal),
isa(Clyde,african),
isa(royal,elephant),
isa(african,elephant),
isa(elephant,gray),*
nisa(royal,gray).

Using the axioms presented in the previous section, *nisa(Clyde,gray)* is yielded, which is the desired result [Sandewall86].

Let us consider a counter-example [Touretzky87], which has the same structure as the above. And suppose a chaplain's non-beer-drinking is less certain than a man's beer-drinking. In this case, we assert *nisa*(chaplain,beerdrinker)* and *isa(man,beerdrinker)* instead of *nisa(chaplain,beerdrinker)* and *isa*(man,beerdrinker)*. Then the axioms yield *isa(George,beerdrinker)*. These examples show that in our system the result of inheritance depends not only on the hierarchical structure, but also on the link type.

(2) *Nixon Diamond*





This is the case where ambiguity arises in multiple inheritances. Suppose the following axioms:

isa(Nixon,republican),
isa(Nixon,quaker),
nisa(republican,pacifist),*
isa(quaker,pacifist).*

Then Nixon's pacifism is ambiguous, that is, *isa*(Nixon,pacifist)* holds in one extension and *nisa*(Nixon,pacifist)* holds in the other. To make the problem more complicated, consider the additional information:

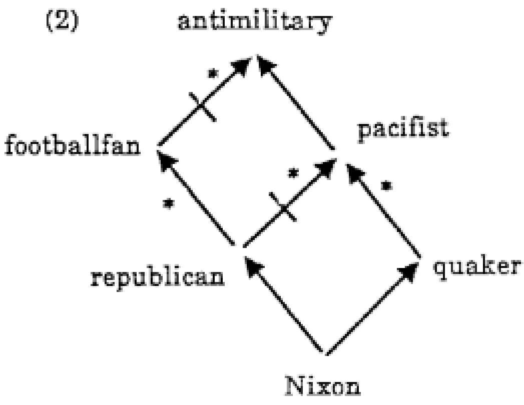
isa(republican,footballfan),*
nisa(footballfan,antimilitary),*
isa(pacifist,antimilitary).

Then we have two extensions; one implies that Nixon is a quaker, republican, footballfan, pacifist and antimilitary, while the other implies that Nixon is a quaker, republican, footballfan and non-pacifist. Note that Nixon cannot inherit non-antimilitary through footballfan via republican, since it uses transitivity through *isa** to *nisa** which is not permitted in the axioms. By changing one of the default *isa* into strict *isa*, however, Nixon's non-antimilitary becomes inheritable using defeasible transitivities and we have three extensions in all.[5]

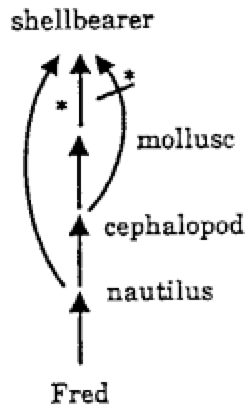
(3) *Shellbearers*

[5]These conform with Touretzky's credulous reasoner.

(2)



(3)



This is a typical situation where duplicate exceptions happen. The hierarchy is represented as:

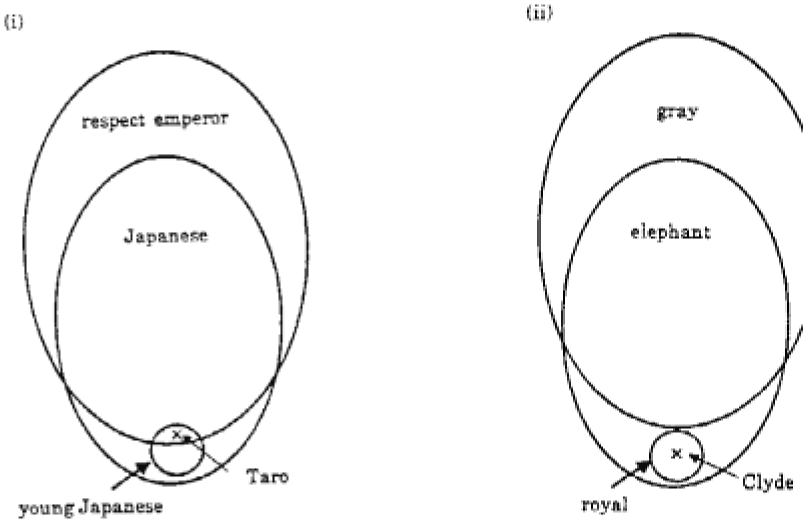
isa(Fred,nautilus),
isa(nautilus,cephalopod),
isa(cephalopod,mollusc),
isa(mollusc,shellbearer),*
nisa(cephalopod,shellbearer),*
isa(nautilus,shellbearer).

Then we obtain *isa(Fred,shellbearer)*. (Though *isa*(Fred,shellbearer)* is also obtained by inheritance from mollusc, it is consistent and subsumed by *isa(Fred,shellbearer)*).

Next, suppose one more fact, *isa(Dick,cephalopod)*, then the above axioms yield not only *nisa*(Dick,shellbearer)* but also *isa*(Dick,shellbearer)*. This is the case which [Etherington83] has pointed out as a fault of [Fahlman79]. However, consider the following case:

isa(Taro,youngJapanese),
isa(youngJapanese,Japanese),
isa(Japanese,respectemperor),*
nisa(youngJapanese,respectemperor),*

This is the same structure as the hierarchy of Dick, cephalopod, mollusc, and shellbearer. (Add imperial-offspring instead of nautilus.) In this situation, we are less certain that Taro does not respect the emperor, since it is a relative problem between the people who respect emperor among Japanese and who do not among young Japanese. Figure (i) shows one



possibility, where $Taro \in respectemperor$ holds. Figure (ii), on the other hand, shows another situation where $Clyde \notin Gray$ always holds.

4 Discussion

We have presented a theory of nonmonotonic inheritances using strict and defeasible transitivities over the strict and default *isa* relations. This is a natural extension of transitive inheritance and gives sound and intuitive semantics for nonmonotonic inheritance reasoning.

Compared with other approaches, Touretzky and Sandewall do not distinguish strict and default *isa* links, which causes the problem for treating example (1). Etherington has distinguished these two links and explicitly represented exceptions by using the exception link. The lack of the exception links in our theory caused the ambiguity about Dick's shellbearing. As is mentioned in example (3), however, the addition of exception links is not straightforward, and needs more careful consideration. Briefly looking at other approaches, [Horty88] also uses strict and defeasible links in his *skeptical* theory, [Padgham88] uses four link types between the *core type* and *default type* in his lattice representation, and [Haugh88] introduces four types of *abnormalities* in his circumscriptive approach. These elaborate studies, however, seem to make the problem more complicated.

The advantage of our theory is its simplicity and clarity, which will facilitate implemention in future work.

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